Απαντήσεις στη τριγωνομετρία Β' Λυκείου

ΘΕΜΑ Α

Α1. Σχολικό σελ 60

Α2 i) Λάθος

ii) Σωστό $\quad \dfrac{\varepsilon\varphi^2\theta}{1+\varepsilon\varphi^2\theta} = \dfrac{\frac{\eta\mu^2\theta}{\sigma\upsilon\nu^2\theta}}{1+\frac{\eta\mu^2\theta}{\sigma\upsilon\nu^2\theta}} = \dfrac{\frac{\eta\mu^2\theta}{\sigma\upsilon\nu^2\theta}}{\frac{\sigma\upsilon\nu^2\theta+\eta\mu^2\theta}{\sigma\upsilon\nu^2\theta}}$

$= \eta\mu^2\theta$

iii) Σωστό

iv) Λάθος

v) Λάθος

ΘΕΜΑ Β

Β1 i) $(2\sigma\upsilon\nu x + 1)\cdot(\varepsilon\varphi x - \sqrt{3}) = 0$

$2\sigma\upsilon\nu x + 1 = 0$ ή $\varepsilon\varphi x = \sqrt{3}$ $\quad x \neq k\pi + \frac{\pi}{2}$

$\sigma\upsilon\nu x = -\frac{1}{2}$ $\quad \varepsilon\varphi x = \varepsilon\varphi\frac{\pi}{3}$

$\sigma\upsilon\nu x = \sigma\upsilon\nu\left(\pi - \frac{\pi}{3}\right)$ $\quad \boxed{x = k\pi + \frac{\pi}{3}}$ $k \in \mathbb{Z}$

$\sigma\upsilon\nu x = \sigma\upsilon\nu\frac{2\pi}{3}$

$\boxed{x = 2k\pi \pm \frac{2\pi}{3}}$ $k \in \mathbb{Z}$

Περιορισμοί : $x \neq k\pi$, $x \neq k\pi + \frac{\pi}{2}$, $k \in \mathbb{Z}$

ii) $\eta\mu x \cdot \sigma\varphi x - \sigma\upsilon\nu x - \sigma\upsilon\nu x \cdot \sigma\varphi x + \eta\mu x = 0$

$\sigma\varphi x(\eta\mu x - \sigma\upsilon\nu x) + (\eta\mu x - \sigma\upsilon\nu x) = 0$

$(\eta\mu x - \sigma\upsilon\nu x) \cdot (\sigma\varphi x + 1) = 0$

$\eta\mu x - \sigma\upsilon\nu x = 0$ ή $\sigma\varphi x + 1 = 0$

$\eta\mu x = \sigma\upsilon\nu x$ $\quad$ $\sigma\varphi x = -1$

$\sigma\varphi x = \sigma\varphi\left(-\frac{\pi}{4}\right)$

$\frac{\eta\mu x}{\sigma\upsilon\nu x} = 1$ $\quad$ $\boxed{x = k\pi - \frac{\pi}{4}}$ $k \in \mathbb{Z}$

$\varepsilon\varphi x = 1$

$\boxed{x = k\pi + \frac{\pi}{4}}$ $k \in \mathbb{Z}$

iii) $3\eta\mu^2 x - 8\eta\mu x + 4 = \sigma\upsilon\nu^2 x$

$3\eta\mu^2 x - 8\eta\mu x + 4 - \sigma\upsilon\nu^2 x = 0$

$3\eta\mu^2 x - 8\eta\mu x + 4 - 1 + \eta\mu^2 x = 0$

$4\eta\mu^2 x - 8\eta\mu x + 3 = 0$

Θέτω $\eta\mu x = \omega$.

$4\omega^2 - 8\omega + 3 = 0$

$\Delta = 64 - 48 = 16$

$\omega_{1,2} = \frac{8 \pm 4}{8} \rightarrow \frac{12}{8} = \frac{3}{2}$, $\frac{4}{8} = \frac{1}{2}$

$\eta\mu x = \frac{3}{2}$ αδύνατη αφού $-1 \leq \eta\mu x \leq 1$

· $\eta\mu x = \frac{1}{2} \Leftrightarrow \eta\mu x = \eta\mu\frac{\pi}{6}$

$$\boxed{x = 2k\pi + \frac{\pi}{6}} \quad \text{ή} \quad \boxed{x = 2k\pi + \frac{5\pi}{6}} \quad k \in \mathbb{Z}$$

B2

A. $\frac{\sigma\upsilon\nu x}{1-\eta\mu x} + \frac{\sigma\upsilon\nu x}{1+\eta\mu x} =$

$$\frac{\sigma\upsilon\nu x(1+\eta\mu x) + \sigma\upsilon\nu x\ (1-\eta\mu x)}{(1-\eta\mu x)(1+\eta\mu x)} =$$

$$= \frac{\sigma\upsilon\nu x + \cancel{\sigma\upsilon\nu x\,\eta\mu x} + \sigma\upsilon\nu x - \cancel{\sigma\upsilon\nu x\,\eta\mu x}}{1-\eta\mu^2 x}$$

$$= \frac{2\sigma\upsilon\nu x}{\sigma\upsilon\nu^2 x} = \frac{2}{\sigma\upsilon\nu x}$$

B. από Α ερώτημα έχουμε

$$\frac{2}{\sigma\upsilon\nu x} = \frac{4}{\sqrt{3}} \Leftrightarrow 4\sigma\upsilon\nu x = 2\sqrt{3}$$
$$\sigma\upsilon\nu x = \frac{\sqrt{3}}{2}$$

$$\sigma\upsilon\nu x = \sigma\upsilon\nu\frac{\pi}{6}$$

$$x = 2k\pi \pm \frac{\pi}{6} \quad k \in \mathbb{Z}$$

Β3 $\quad \eta\mu\omega = \frac{3}{5} \qquad \frac{\pi}{2} < \omega < \pi$

$$\eta\mu^2\omega + \sigma\upsilon\nu^2\omega = 1$$

$$\sigma\upsilon\nu^2\omega = 1 - \eta\mu^2\omega$$

$$\sigma\upsilon\nu^2\omega = 1 - \frac{9}{25}$$

$$\sigma\upsilon\nu^2\omega = \frac{16}{25}$$

$$\sigma\upsilon\nu\omega = \pm\frac{4}{5}$$

Αφού $\frac{\pi}{2} < \omega < \pi$ τότε $\boxed{\sigma\upsilon\nu\omega = -\frac{4}{5}}$

$$\varepsilon\varphi\omega = \frac{\eta\mu\omega}{\sigma\upsilon\nu\omega} \Leftrightarrow \varepsilon\varphi\omega = \frac{\frac{3}{5}}{-\frac{4}{5}}$$

$$\boxed{\varepsilon\varphi\omega = -\frac{3}{4}}$$

$$\sigma\varphi\omega = \frac{\sigma\upsilon\nu\omega}{\eta\mu\omega} = \frac{-\frac{4}{5}}{+\frac{3}{5}} = -\frac{4}{3}$$

$$\Leftrightarrow \boxed{\sigma\varphi\omega = -\frac{4}{3}}$$

ΘΕΜΑ Γ.

i) $\cdot \eta\mu(\pi-\frac{x}{2}) = \eta\mu\frac{x}{2}$

$\cdot \varepsilon\varphi(17\pi-x) = \varepsilon\varphi(16\pi+\pi-x) = \varepsilon\varphi(\pi-x) = -\varepsilon\varphi x$

$\cdot \sigma\upsilon\nu\left(\frac{11\pi}{2}-\frac{x}{2}\right) = \sigma\upsilon\nu\left(\frac{10\pi}{2}+\frac{\pi}{2}-\frac{x}{2}\right)$

$= \sigma\upsilon\nu(5\pi+\frac{\pi}{2}-\frac{x}{2}) = \sigma\upsilon\nu(4\pi+\pi+\frac{\pi}{2}-\frac{x}{2})$

$= \sigma\upsilon\nu(\pi+\frac{\pi}{2}-\frac{x}{2}) = -\sigma\upsilon\nu(\frac{\pi}{2}-\frac{x}{2}) = -\eta\mu\frac{x}{2}$

$\cdot \sigma\varphi\left(\frac{5\pi}{2}\right) = \sigma\varphi\left(\frac{3\pi}{2}+\frac{\pi}{2}-x\right) = \sigma\varphi(4\pi+\frac{\pi}{2}-x)$

$= \sigma\varphi(\frac{\pi}{2}-x) = \varepsilon\varphi x$ (Η απευθείας με μνημονικό κανόνα!)

$f(x) = -2\eta\mu\frac{x}{2} + (-\varepsilon\varphi x) + (-\eta\mu\frac{x}{2}) + \varepsilon\varphi x$

$f(x) = -3\eta\mu\frac{x}{2}$

ii) $-1 \le \eta\mu\frac{x}{2} \le 1$

$3 \ge -3\eta\mu\frac{x}{2} \ge -3$

$-3 \le -3\eta\mu\frac{x}{2} \le 3$

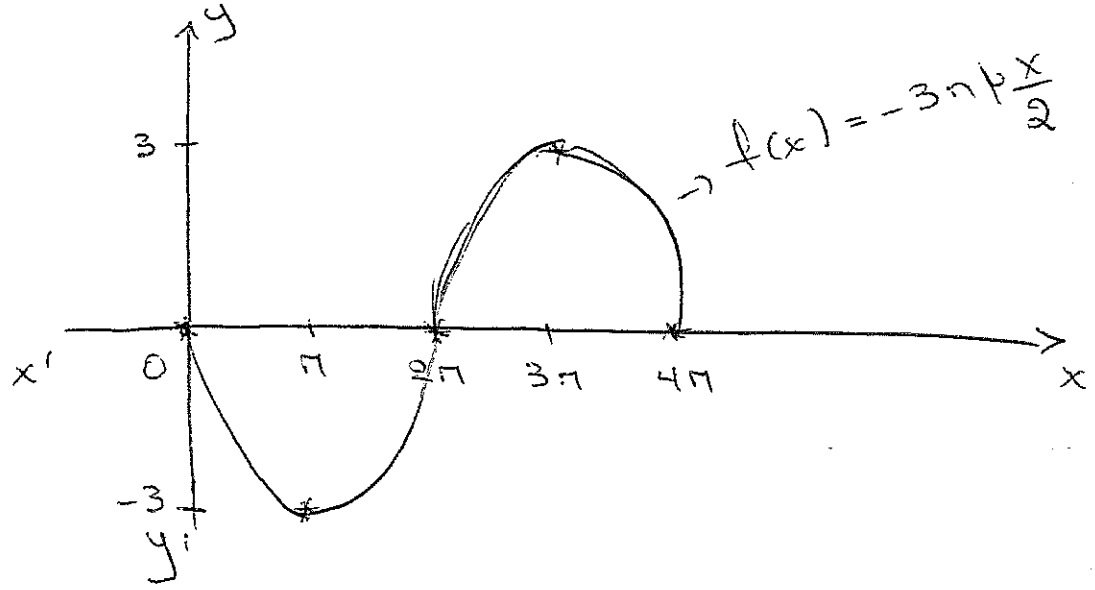
$-3 \le f(x) \le 3$

· Η μεγίστη τιμή είναι 3 και η ελάχιστη τιμή -3.

· Η περίοδος Τ είναι $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

ii)

x	0	π	2π	3π	4π
$\frac{x}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\eta\mu\frac{x}{2}$	0	1	0	-1	0
$f(x)$	0	-3	0	3	0



iv) $f(x) = \frac{3}{2}$

$-3\eta\mu\frac{x}{2} = \frac{3}{2} \Leftrightarrow \eta\mu\frac{x}{2} = -\frac{1}{2}$

$\eta\mu x = \eta\mu\left(-\frac{\pi}{6}\right)$

$\frac{x}{2} = 2k\pi - \frac{\pi}{6}$ ή $\frac{x}{2} = 2k\pi + \frac{7\pi}{6}$

$x = 4k\pi - \frac{\pi}{3}$ ή $x = 4k\pi + \frac{7\pi}{3}$

$x \in (-\pi, 2\pi)$

$-\pi < x < 2\pi$

$-\pi < 4k\pi - \frac{\pi}{3} < 2\pi$

$-\pi + \frac{\pi}{3} < 4k\pi < 2\pi + \frac{\pi}{3}$

$-\frac{2\pi}{3} < 4k\pi < \frac{7\pi}{3}$

$-\frac{2}{12} < k < \frac{7}{12}$

$-\frac{1}{6} < k < \frac{7}{12}$

άρα για $k=0$

έχω $\boxed{x = -\frac{\pi}{3}}$

ή

$-\pi < x < 2\pi$

$-\pi < 4k\pi + \frac{7\pi}{3} < 2\pi$

$-\pi - \frac{7\pi}{3} < 4k\pi < 2\pi - \frac{7\pi}{3}$

$-\frac{10\pi}{3} < 4k\pi < -\frac{\pi}{3}$

$-\frac{10}{12} < k < -\frac{1}{12}$

$-\frac{5}{6} < k < -\frac{1}{12}$

Δεν υπάρχει ακέραιος στο διάστημα